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Seismology of a Sunspot Atmosphere

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Abstract Two competing theories of sunspot oscillations are discussed. It is pointed out that the normal mode (eigenoscillations) theory is in contradiction with a number of observations. The reasons for this are discussed. The revised filter theory of the three-minute sunspot oscillations is outlined. It is shown that the reason for the occurrence of the multi-passband filter for the slow waves is the interference that appears from the multilayer structure of the sunspot atmosphere. In contrast with Zhugzhda and Locans (*Sov. Astron. Lett.* **7**, 25 – 27, 1981) it is shown that along with the Fabry – Perot chromospheric passband the cutoff frequency passband and a number of the high-frequency passbands occur. The effect of the nonlinearity of the sunspot oscillations in the upper chromosphere and the transition region is taken into account. The spectra of the distinct empirical models of the sunspot atmosphere are explored. An example of the interpretation of the sunspot oscillations based on the revised filter theory is presented. Only the filter theory can explain the complicated behavior of the oscillations across the sunspot. The observations provide evidence of the nonuniformity of the sunspot atmosphere.

Keywords Sun: sunspots · Sun: oscillations

A long time has elapsed since the discovery of the three-minute oscillations. Numerous studies of the oscillations have been performed. The story of the discovery and exploration of sunspot oscillations can be found in the review of Bogdan and Judge (2006) devoted to the observational aspects of the sunspot oscillations. We know a lot about the three-minute

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oscillations in the photosphere, chromosphere, and corona of sunspots. But so far no universally accepted theory of the three-minute oscillations exists. There are two competing theories: the filter theory and the theory of eigenoscillations of sunspots. The general theory of the eigenmodes of the atmosphere permeated by a uniform, vertical, magnetic field was developed by Cally and Bogdan (1993) and Bogdan and Cally (1997). This theory is believed to form a foundation for the theory of sunspot oscillations. But there are some contradictions between the concept of sunspot eigenoscillations and observations. It is found by many observers that the three-minute oscillations are not standing waves, as would be the case for normal modes. They are waves that are running from the sunspot photosphere through the chromosphere to corona. Besides, the local seismology of sunspots shows that the waves are going freely into and through the sunspot (Zhao and Kosovichev, 2006). This is again in contradiction with the concept of normal modes since the incident waves have to be captured by the sunspot cavity and re-emitted back after some time defined by the quality of the cavity. Thus, even in the case of a low-quality cavity, the delay of the reradiation of the captured waves has to be about a few periods of the oscillations. Nothing similar to this has been observed. Thus, there is no evidence that the sunspot is a cavity for the three-minute oscillations. This does not mean that the theory of normal modes of the atmosphere permeated by a magnetic field is mistaken. The problem is with the boundary conditions used for the treatment of normal modes. For example, Bogdan and Cally (1997) fixed the horizontal wavenumber. In fact it does mean that the condition of the complete reflection of the waves from the side walls of the sunspot magnetic tube was imposed. But the assumption of complete reflection does not work in subphotospheric layers since the jump of the plasma parameters across the boundary of the sunspot is not large enough to make a strong reflection. There is some concern about the boundary conditions at the bottom of the sunspot since the deep structure of the sunspot is still under discussion. Thus, if there is no trapping of the waves by the sunspot, the only way is to look for the propagation of the waves through the sunspot atmosphere. Zhugzhda and Locans (1981) were the first to explore wave propagation as an alternative to trapping of waves. They assumed that the three-minute oscillations are slow MHD waves, which was confirmed later by many observations. They revealed that the sunspot chromosphere can work as a Fabry – Perot filter for the slow waves. That is why we call this approach the "filter theory." Later on many calculations of slow-wave propagation through the sunspot atmosphere were carried out with modeling of the sunspot atmosphere (Settele, Staude, and Zhugzhda, 2001, and references there). In fact, it was revealed that there are numerous passbands for the slow waves (*i.e.*, the sunspot atmosphere works as a multiband filter for the slow waves). However, interpreting the wave filtering by the sunspot on the basis of the Fabry – Perot effect faces some problems. For example, the calculations and observations sometimes show close spectral peaks whereas the Fabry – Perot filters produce equidistant passbands. Furthermore, B.W. Lites and T.J. Bogdan (private communications) have raised the issue of the effect of the cutoff frequency, which can produce the spectral peak. Zhugzhda (2007) showed that the Fabry – Perot effect is not the only effect that affects slow-wave propagation in the sunspot atmosphere. The current paper is devoted to the complete revision of the filter theory. Another crucial point under discussion in this paper is whether the linear theory of wave propagation can be applied to the treatment of the three-minute oscillations since it was revealed that three-minute oscillations become nonlinear in the upper chromosphere and transition region (see, for example, Centeno, Collados, and Bueno, 2006). Finally, the interpretation of the observations of the three-minute oscillations in the framework of the filter theory is presented.

1. Basics of Slow-Wave Propagation

The propagation of MHD waves in a conducting atmosphere permeated by a vertical uniform magnetic field is governed by the following set of the equations (Cally and Bogdan, 1993):

$$\left[c_{\mathrm{A}}^2\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\omega^2-k_\perp^2\left(c_{\mathrm{S}}^2+c_{\mathrm{A}}^2\right)\right]v_\perp=-\mathrm{i}k_\perp\left(c_{\mathrm{S}}^2\frac{\mathrm{d}}{\mathrm{d}z}-g_{\mathrm{eff}}\right)v_\parallel, \tag{1}$$

$$\begin{aligned}&\left[c_{\mathrm{S}}^2\frac{\mathrm{d}^2}{\mathrm{d}z^2}-\left(\gamma g_{\mathrm{eff}}+c_{\mathrm{S}}^2\frac{\mathrm{d}\ln\gamma}{\mathrm{d}z}\right)+\omega^2-c_{\mathrm{S}}^2\frac{\mathrm{d}g_{\mathrm{eff}}}{\mathrm{d}z}\right]v_\parallel\\&\quad=-\mathrm{i}k_\perp\left(c_{\mathrm{S}}^2\frac{\mathrm{d}}{\mathrm{d}z}-g_{\mathrm{eff}}(\gamma-1)-c_{\mathrm{S}}^2\frac{\mathrm{d}\ln\gamma}{\mathrm{d}z}\right)v_\perp,\end{aligned} \tag{2}$$

$$g_{\mathrm{eff}}=g-\frac{1}{\rho}\frac{\mathrm{d}p_{\mathrm{turb}}}{\mathrm{d}z}, \tag{3}$$

where c_{A} and c_{S} are the Alfvén and sound speeds, respectively, $v_\parallel$ and $v_\perp$ are the vertical and horizontal components, respectively, of the plasma velocity, p_{turb} is the turbulent pressure, and g_{eff} is the effective gravity taking into account the effect of turbulent pressure on the hydrostatic equilibrium of the atmosphere. The effect of turbulent pressure is essential in the chromosphere. The contribution from the turbulent pressure in maintaining hydrostatic equilibrium must be taken into account, because, otherwise, the energy-flux conservation law for waves is violated (Mihalas and Toomre, 1981). There is no surprise that this effect was not taken into account by the theory of normal modes since it is easy to miss the violation of energy conservation in this case. Another issue is with the treatment of the wave propagation. The effect of turbulent pressure was taken into account in all our explorations of the slow-wave propagation in the sunspot atmosphere.

Two coupled Equations (1) and (2) describe the propagation and coupling of the fast and slow waves. The right-hand parts of the equations define the wave coupling. In accordance with the general theory of MHD wave coupling (Zhugzhda, 1979; Zhugzhda and Dzhalilov, 1982) the coupling of the waves is negligible in the low-β plasma of the solar chromosphere. Moreover, the fast waves of three-minute period are evanescent in the chromosphere, temperature minimum, and upper photosphere of the sunspots whereas the slow waves are running waves. Thus, there is no question that the three-minute oscillations are slow waves since three-minute oscillations are running waves. The coupling between running and evanescent waves is very weak.

In the case of weak coupling of slow and fast waves the right-hand sides of Equations (1) and (2) can be dropped. The propagation of the slow waves in this case is governed by the approximate equation, which was obtained for the first time by Syrovatskii and Zhugzhda (1968):

$$\left[c_{\mathrm{S}}^2\frac{\mathrm{d}^2}{\mathrm{d}z^2}-\left(\gamma g_{\mathrm{eff}}+c_{\mathrm{S}}^2\frac{\mathrm{d}\ln\gamma}{\mathrm{d}z}\right)+\omega^2-c_{\mathrm{S}}^2\frac{\mathrm{d}g_{\mathrm{eff}}}{\mathrm{d}z}\right]v_\parallel=0, \tag{4}$$

$$g_{\mathrm{eff}}=g-\frac{1}{\rho}\frac{\mathrm{d}p_{\mathrm{turb}}}{\mathrm{d}z}. \tag{5}$$

Only the effective gravity was included later. The neglect of wave coupling is possible for the photosphere in the case when the horizontal wavelength of the slow waves is about the sunspot diameter. We specifically rewrite this equation in the final form because there is serious confusion in connection with it. Equation (4) looks identical to the equation for vertically

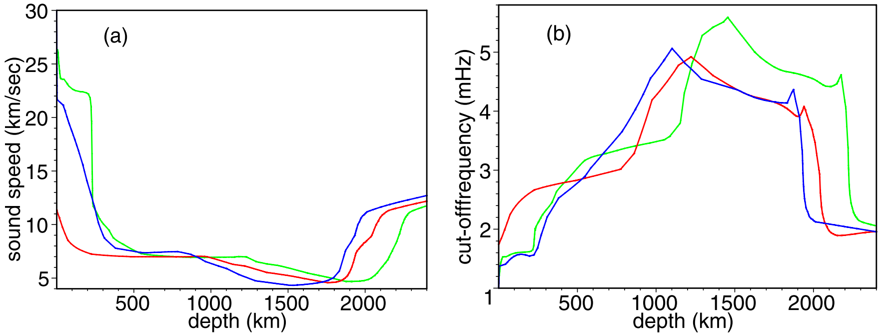

Figure 1 The sound speed (a) and cutoff frequency (b) as a function of depth for the empirical models Lites (green curves), Maltby (red curves), and Staude (blue curves).

propagating plane acoustic waves since there is no dependence on the horizontal wavenumber ($k_\perp$). This gives rise to an unfortunate mistake that is commonly encountered in the literature when no distinction is made between acoustic and slow magneto-acoustic-gravity waves. Sometimes the slow waves are called acoustic waves (see, for example, Brynildsen *et al.*, 1999; Brekke *et al.*, 2003). This is not correct. In fact, the slow waves propagate independently along each magnetic field line. Thus, Equation (4) can be applied separately to the treatment of the propagation of the slow waves along the separate field lines of the sunspot magnetic field if ones replaces the simplified model of the uniform vertical magnetic field by the diverging field or even by the real magnetic field of the individual sunspots. However, it is reasonable to mention that if the sunspot atmosphere is nonuniform (*i.e.*, the temperature and pressure of the plasma of the adjacent field lines are different), the slow waves propagate with a velocity less than the sound speed. If the sunspot atmosphere consist of thin flux tubes, the slow waves propagate with a tube speed $c_\mathrm{A} c_\mathrm{S}/\sqrt{c_\mathrm{A}^2 + c_\mathrm{S}^2}$, which is less than the sound speed (Zhugzhda and Goossens, 2001). The distinction between tube and sound speeds is negligible in the chromosphere but can be essential in the photospheric and subphotospheric layers of the sunspot.

One additional shortcoming of the filter theory, which has yet to be overcome, is the neglect of nonadiabatic effects. Taking these effects into account leads to a change of the phase velocity and the appearance of wave absorption.

2. Peculiarities of Sunspot Atmosphere

There are few empirical models of sunspot atmosphere. The different models show marked distinctions. Figures 1(a) and 1(b) show the sound speed and the cutoff frequency for the three well-known models of sunspot atmosphere, namely, the Staude (1981), Maltby *et al.* (1986), and Lites and Skumanich (1982) models. Differences among the empirical models based on observations can appear for different reasons, namely, differences among the individual sunspots, the inaccuracy of the observations, and the shortcomings of the modeling procedure. Seismology of the sunspot atmosphere based on observations of the three-minute oscillations can help elucidate the reason for the distinctions among the empirical models of the sunspot atmosphere. In connection with slow-wave propagation, two main features of the atmosphere are important: The first is the occurrence of the cutoff frequency, which is responsible for the low-frequency limit of the wave spectrum. The second is the nonmonotonic dependence of the temperature on altitude in the sunspot atmosphere, which causes the interference of waves. Only in the case of a nonmonotonic temperature profile does the transmission function of the slow waves display passbands. The waves running through the atmosphere with monotonic temperature gradient undergo the same reflection at all levels. In the case of a nonmonotonic temperature profile, the largest reflection of slow waves occurs at the levels of the maximum temperature gradient. This is a crucial point for the filter theory since the interference of the waves reflected from these levels can increase or decrease the transmission of the waves. The temperature profile for the different empirical models shown on Figure 1 is nonmonotonic. It has to be the enhancement of the reflection at the boundaries between the chromosphere, temperature minimum, photosphere, and subphotospheric layers. The wavelength of three-minute oscillations in the sunspot atmosphere is in the range $(1-2) \times 10^3$ km and the thickness of the transitions layers between the temperature minimum and temperature plateau does not exceed a few hundred kilometers. Thus, rather complicated interference of the slow waves occurs in the sunspot atmosphere. The Fabry – Perot effect predicted by Zhugzhda and Locans (1981) is only one among others. This is the reason why the interpretation of slow-wave propagation through the sunspot atmosphere (see Settele, Staude, and Zhugzhda, 2001, and references there) has to be revised. The use of empirical models of the sunspot atmosphere for the exploration of the nature of the distinct interference effect meets with difficulties since the parameters of the sunspot atmosphere, namely, the thickness and temperature of the different layers of the atmosphere, need to be varied. Thus, we arrive at the idea of exploring first a rather crude four-layer model of the sunspot atmosphere to understand the basics of the interference effect in a multilayer atmosphere with a temperature minimum.

3. Four-Layer Model of the Sunspot Atmosphere

To explore the basics of the slow-wave propagation the simple four-layer model of the sunspot atmosphere shown in Figure 1(a) has been used. The four isothermal layers correspond to the corona at temperature T_4, the chromosphere at temperature T_2, the temperature minimum at temperature T_1, and the photosphere at temperature T_3. The dimensionless thickness ($D = d/H_1$) is measured in units of the pressure scale height H_1 of the temperature minimum. Two scenarios of wave propagation are explored, namely, for frequencies below and above the cutoff frequency of the temperature minimum The discontinuities between the isothermal layers produce a strong reflection in comparison with the real sunspot atmosphere. But this simple model helps us to explore the basic effects.

3.1. Fabry – Perot Chromospheric Filter

If the frequency of the slow waves is less than the cutoff frequency of the temperature minimum, the waves undergo strong reflection from the temperature minimum, and only a small fraction penetrates to the chromosphere by a tunnel effect (see the bottom scheme of arrows in Figure 1(a)). After partial reflection from the corona the waves penetrate back through the temperature minimum to the photosphere, where, as shown in Figure 2(a), they interfere with the waves reflected from the temperature minimum. In the case of destructive interference of the waves the reflection from the temperature minimum decreases and the transmission of waves to the solar corona increases at the frequency of the chromospheric resonance. Thus, the four-layer atmosphere works as a Fabry – Perot chromospheric filter for the slow waves.

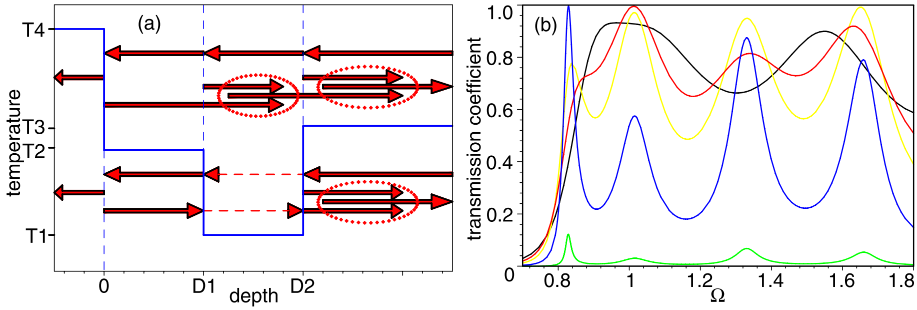

Figure 2 (a) The four-layer model of the sunspot atmosphere. The temperature profile as a function of the dimensionless depth is shown by the solid blue curve. Shown are the two scenarios of wave propagation, namely, for frequencies below (bottom) and above (top) the cutoff frequency of the temperature minimum. (b) The transmission coefficient for the distinct values of the amplitude reduction of the waves approaching the temperature minimum from the upper layers of the sunspot atmosphere $R = 1.0$ (green), 0.87 (blue), 0.55 (yellow), 0.29 (red), and 0.09 (black).

The frequency of the Fabry – Perot passband equals the frequency of the chromospheric resonance. But there is a crucial difference from classical Fabry – Perot filters. Unlike classical Fabry – Perot filters, four-layer atmospheres can work only as a single-band filter, whereas classical Fabry – Perot filters have a number of the equidistant passbands. The reason for the distinction is the strong frequency dependence of tunneling of the waves through the temperature minimum. Almost total transmission of slow waves through the sunspot chromosphere occurs when the reflection of the slow waves from the temperature minimum and the solar corona are equal each other, and the reflected waves cancel each other. But this condition is met only for a single frequency (*i.e.*, for the special choice of the thickness and temperature of the chromosphere). If the frequency of the chromospheric resonance decreases below this optimal frequency the transmission of waves decreases. However, the frequency of the Fabry – Perot passband cannot exceed the cutoff frequency of the temperature minimum owing to a decrease of the reflection from the temperature minimum. Thus, there is some range of frequencies where the chromospheric Fabry – Perot filter is effective. This range is so small that as a rule only one possible passband of the Fabry – Perot filter falls in the range. Moreover, if this solitary passband is near the edge of this range, the transmission of the passband can be so small that it does not produce a noticeable peak in the spectrum of the sunspot oscillations. Thus, we arrive at the conclusions that *i*) the chromospheric Fabry – Perot filter can be responsible for only one peak in the spectrum of three-minute oscillations, *ii*) the frequency of this peak has to be less than the cutoff frequency of the temperature minimum, and *iii*) the Fabry – Perot filter does not produce an observable peak in the spectrum of the oscillations in some cases, when the frequency of the chromospheric resonance does not fall within the prescribed range of frequencies. Therefore, the interpretation of the entire spectrum of three-minute oscillations by the Fabry – Perot filter effect is not possible.

The green curve in Figure 2(b) is the transmission coefficient for the slow waves running through the four-layer atmosphere. The transmission coefficient equals the ratio of the transmitted to incident energy fluxes of the slow waves. The calculations are performed for the following choice of sound-speed ratios: $c_{01}/c_{02} = 0.6$, $c_{03}/c_{02} = 2$, and $c_{04}/c_{02} = 14$, which roughly correspond to the ratios of the sound speeds of the corona, chromosphere, temperature minimum, and photosphere of the sunspots. The green curve is calculated for $D_1 = 2.5$, $D_2 = 12$. The transmission coefficient is a function of the dimensionless frequency $\Omega = \omega H_2/c_{02}$. The transmission coefficient is rather small because the reflection from the boundaries between layers is rather strong. The transmission band of the lowest frequency in the plot of Figure 2(b) is the Fabry – Perot passband.

3.2. Cutoff Passband and High-Frequency Passbands

The next passband appears at the cutoff frequency of the temperature minimum. A first naive glance would indicate that all of the waves of frequencies above the cutoff frequency of the temperature minimum propagate freely through the temperature minimum. But this is not the case since the waves undergo reflection from the boundaries of the temperature minimum. This effect is known in quantum mechanics as over-barrier reflection. The transmission function shown by the black curve in Figure 2(b) is calculated for the case of the three-layer atmosphere, when the corona is absent. The transmission of the slow waves for frequencies above the cutoff frequency is not constant. The interference bands appear from the interference of the waves reflected from the two boundaries of the temperature minimum. The frequency interval between them is defined by the travel time of waves through the temperature minimum. In the case of the sunspot atmosphere, the process is more complicated because of reflection from the corona. The scenario of wave propagation for frequencies above the cutoff frequency of the temperature minimum is shown in Figure 2(a) by the upper scheme of arrows. In this case, interference occurs in the temperature minimum and the photosphere. Numerous transmission bands for frequencies above the cutoff frequency appear because of this interference (Figure 2(b)). The mechanism of the origin of these passbands is similar to the antireflection effect for a multilayer coating, well-known in the optics and acoustics (Brekhovskikh and Godin, 1990). The sunspot atmosphere consists of a few layers, namely, the chromospheric temperature plateau, the temperature minimum, and, possibly, the photosphere. The sunspot atmosphere differs significantly from the optical multilayer coating in that it does not consist of exact quarter-wavelength layers for special values of the refraction index. Besides, the boundaries between the layers are smoothed out. As a consequence, the passbands are not equidistant and their transmission is not equal.

It should be emphasized that the frequency of the cutoff passband is not exactly equal to the cutoff frequency even in the simplest case of the ideal three-layer atmosphere when there is no reflection from the corona (Zhugzhda, 2007). But the frequency of the cutoff passband is subject to the adjacent interference passbands, namely, the Fabry – Perot passband and the first high-frequency passband. When they are located near the cutoff passband it is shifted to high or low frequencies. Thus, the frequency of the cutoff band coincides with the cutoff frequency with high accuracy only when it is far enough away from the adjacent passbands.

3.3. The Effect of Nonlinearity

In recent years it has become clear that the oscillations in the upper chromosphere and transition region above the sunspots are nonlinear. They have a sawtooth shape (*i.e.*, the fronts become steeper and shock waves are formed). This raises the question as to whether the filter theory based on the linear theory of wave propagation is applicable to the problem. In fact, the formation of the interference passbands for slow waves propagating through the sunspot atmosphere does not depend directly on nonlinear effects in the upper atmosphere, since the wave interference takes place in the lower atmosphere, more specifically, in the temperature minimum and the photosphere where the wave amplitude is low, and the applicability of the linear theory is beyond any doubt. However, the nonlinear effects indirectly affect the formation of the interference transmission bands. The nonlinearity leads to the nonlinear absorption of the waves, which causes a reduction of the waves coming from the upper layers to the regions where the interference takes place. The reduction of the wave amplitudes

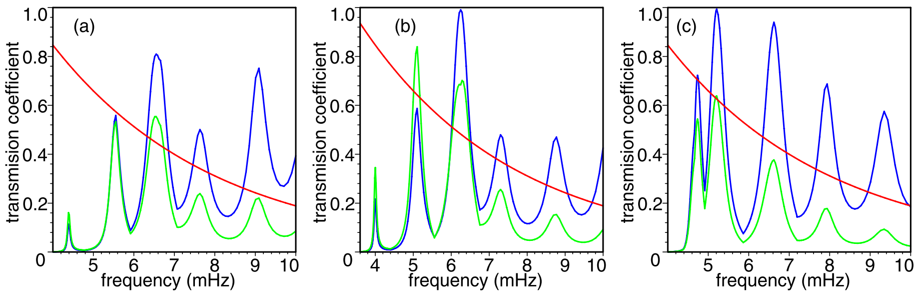

Figure 3 Model spectra (green), transmission functions (blue), and the decline of the spectrum of the incident noise in arbitrary units (red) for sunspot models of Lites (a), Maltby (b), and Staude (c).

affects the transmission of the waves through the atmosphere because the transmission of the passbands is defined by the ratio of the amplitudes of the interfering waves. The frequencies of the passbands can be shifted only by the change of the travel time of the waves from the upper chromosphere to the temperature minimum. However, the frequency shifts of passbands caused by the nonlinearity is hardly significant since the linear and nonlinear slow waves are moving approximately with the sound speed.

Unfortunately, it is difficult to evaluate the nonlinear absorption coefficient as a function of the amplitude of the incident waves. But it is simple to explore the effect of the nonlinearity by assuming different values for the reduction of the amplitude of the waves arriving at the temperature minimum from the upper layer of the sunspot atmosphere. The colored curves in Figures 2(b) are the transmission coefficients for the distinct values of amplitude reduction R that cover the entire range of possible absorptions from its absence (green curve) to complete absorption (black curve). A small nonlinear absorption makes the Fabry – Perot chromospheric filter almost perfect (see the blue curve on Figure 2(b)). Further increasing the nonlinear absorption makes the Fabry – Perot filter less and less transparent. It is interesting that the nonlinear absorption affects the cutoff passband as well. The proximity of the Fabry – Perot passband to the cutoff passband leads to involvement of interference effects in the formation of the cutoff passband. The nonlinear absorption leads, as a rule, to an increase in transmission for all high-frequency passbands. However, the complete absorption of the waves in the upper atmosphere changes drastically the transmission function (see the black curves on Figure 2(b)) since the interference in the temperature minimum disappears. This changes the frequency intervals between the high-frequency passbands. The frequency interval between the high-frequency interference passbands is defined by the travel time through the layers involved in their formation. In the case of a four-layer atmosphere, only two layers are involved in the formation of the high-frequency passbands. If the difference between travel times in the layers is significant, then the high-frequency passbands are not equidistant, and their transmission varies from band to band. The entire travel time through the layers defines in this case the mean interval between passbands. Absorption resulting from nonadiabaticity of the waves has to affect the transmission of waves, as well.

With knowledge of the interference effects, we arrive at an exploration of the filter properties of the empirical models of sunspot atmosphere, which without question provides a better description of the sunspot atmosphere than the four-layer atmosphere.

4. Spectra of Oscillations for Empirical Models of Sunspots

One should not directly compare the transmission functions with the spectrum of the oscillations in sunspots. To obtain the spectrum, we must multiply the transmission function by the spectrum of the waves that come from the subphotospheric layers. This spectrum is unknown, although it is clear that it must decrease rapidly with frequency. In addition, nonlinear wave absorption has to be taken into account. However, nonlinear effects are essential only for frequencies at which the wave amplitude is large enough. Consequently, nonlinear absorption must be taken into account only when the transmission function is calculated within the main transmission bands and must be disregarded at frequencies that correspond to the minima of the transmission function and at high frequencies. For simplicity, we assume in the calculations presented in Figure 2(b) that the nonlinear absorption does not depend on frequency.

To get an idea of how the oscillation spectra look for the empirical models, the calculations were performed under the assumption that the spectrum of the incident waves decreased exponentially with frequency and that the nonlinear absorption was proportional to the wave amplitude. Examples of such model spectra for the Lites, Maltby, and Staude empirical models are shown in Figures 3(a), 3(b), and 3(c). In general, the model spectra shown in these figures are similar to what is actually observed in the sunspots. The spectra for different sunspot models differ markedly. This is not surprising since, as has already been noted, the models themselves differ greatly.

The lowest frequency peaks in the spectra of the three sunspot models appear because of the Fabry – Perot chromospheric filter and their frequencies are determined by the thickness and temperature of the chromospheric plateau. The distinction between the frequencies of these peaks is most pronounced owing to the difference in the temperature plateau thickness for the three models. The oscillation spectrum for the Staude model exhibits another feature of the chromospheric interference filter that distinguishes it from an ordinary Fabry – Perot filter. As the cutoff frequency of the temperature minimum is approached by the Fabry – Perot passband, the chromospheric resonance frequency increasingly deviates toward lower frequencies from the frequency of the wave whose half-wavelength fits into the thickness of the temperature plateau. This is because the wave field in this case extends to an increasingly larger part of the temperature minimum. In this case, as previously mentioned, the cutoff passband shifts to higher frequencies because of the influence of interference effects. Thus, in the case of the appearance of the double peak, the nearness of the cutoff frequency to the chromospheric resonance frequency makes it difficult to use them for estimating the thickness of the temperature plateau and the cutoff frequency.

The frequency interval between the high-frequency interference passbands does not correspond to the travel time through the chromosphere and temperature minimum. They are located closer to each other than this travel time suggests. The frequency interval between these bands corresponds to the travel time through almost the entire sunspot atmosphere from the corona to the deep layers of the photosphere. Thus the three layers are involved in the formation of the high-frequency passbands. The lower layer is located between the temperature minimum and the deep layers of the photosphere where the temperature gradient increases. There is no surprise that it is not possible to reproduce perfectly the filter function of the empirical models by the choice of parameters of the four-layer atmosphere. The problem lies not only with the number of layers but with the strong discontinuities between layers, which leads to strong reflections of waves. This becomes clear from a comparison of the filtering functions for the four-layer atmosphere (the green curve of Figure 3(a)) with the empirical atmosphere (see Figure 2 in Settele, Staude, and Zhugzhda, 2001) in the case when

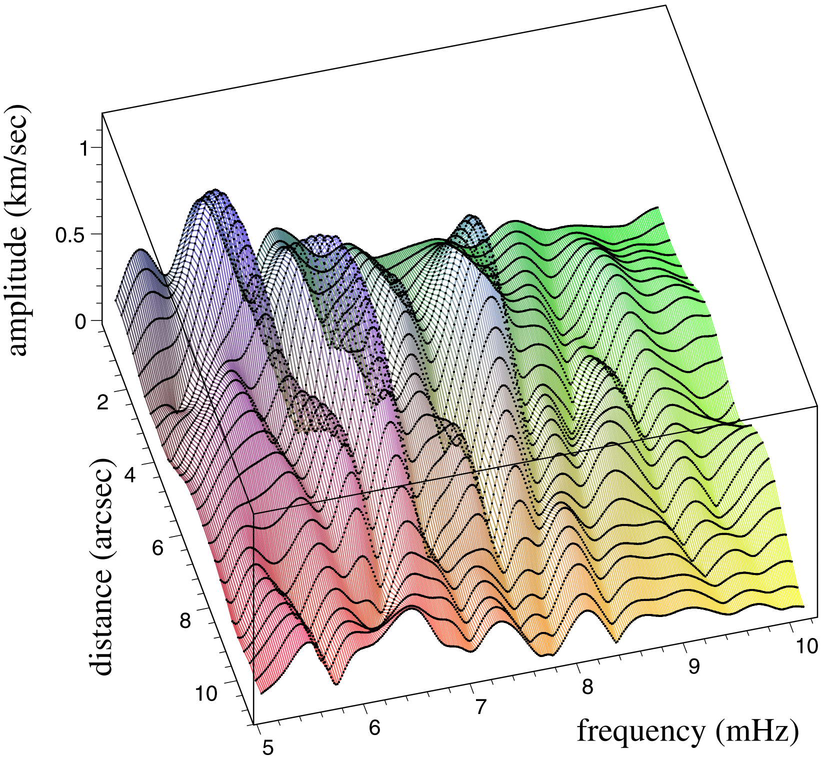

Figure 4 The amplitude spectrum of the chromospheric oscillations in the sunspot as a function of the distance along the slit.

nonlinear absorption of waves is absent. In general, the transmission of slow waves through the empirical atmosphere exceeds the transmission through the four-layer atmosphere. Thus, the four-layer atmosphere is just a tool to explore the interference effects in a sunspot atmosphere. Now we arrive at the next step of the exploration, that is, whether the observations can be explained in terms of the revised filter theory.

5. Interpretation of Observations by the Filter Theory

R. Centeno, M. Collados, and J. Trujillo Bueno provided me with the results of data processing of their unique observations of sunspot oscillations, making the current exploration of sunspot oscillations possible. (The details of the observations and the processing can be found in Centeno, Collados, and Bueno (2006).) The observations of the sunspot oscillations by Centeno, Collados, and Bueno (2006) were taken in the He I 10830 Å multiplet. In addition to the analysis of Centeno, Collados, and Bueno (2006) the variation of the spectrum of the oscillations along the slit is explored.

The goal of the current exploration is to check whether the sunspot oscillates as a whole. The theory of normal modes of the sunspots is based, in fact, on the assumption that the sunspot oscillates as a single entity. This is one of the reasons why it is usual to look for oscillations of the entire sunspot. Of course, the inhomogeneity of the sunspot is not included in the theoretical models for reasons of simplicity. However, the inhomogeneity of the sunspot atmosphere can lead only to the inhomogeneity of the amplitude of the normal modes across the sunspot but it cannot give rise to the frequency variations across the sunspot. It is quite another matter with the filter theory since, as already pointed out, the slow waves propagate independently along each magnetic field line. Thus, the distinctions of the temperature stratification along different field lines causes the changes of the passband frequencies across the sunspot. The distinctions of the temperature structure can appear because of the inhomogeneity of sunspot atmosphere as well as because of the divergence of the magnetic-field lines, which cause the distinctions of the temperature scale along the distinct field lines. Thus, it is difficult to assume that the transmission function for the slow waves is the same over the sunspot umbra.

To look for the variations of the spectrum of three-minute oscillations, the spectrum of sunspot oscillations for each of 26 points along the slit was obtained. The dependence of the spectrum on distance along the slit is presented in Figure 4. There is no doubt that the passband frequencies are not constant over the sunspot umbra. To trace the changes in the spectrum along the slit the dependence of the peak frequencies in the spectrum on the distance along the slit is presented in Figure 5(a). The frequency of the cutoff passband is not constant across the sunspot. It is about 5.9 mHz and almost constant along the slit in the range $0'' - 3''$. Then at the point $4''$ the high-frequency passband splits away from the cutoff passband. The frequency of the cutoff band shifts to lower frequencies because of the influence of the adjacent high-frequency passband. It varies in the range between 5.5 and 5.8 mHz. Near the far end of the slit, the high-frequency band disappears, and the frequency of the cutoff passband is about 5.9 mHz as it is on the opposite end of the slit. The cutoff frequency varies inversely with the sound speed. But it is difficult to believe that the temperature in the temperature minimum increases in the middle coolest parts of the sunspot umbra except in the case of the light bridges. Thus, the decrease of the frequency of the cutoff passband occurs mostly likely from the rearrangement of the high-frequency passbands. The first high-frequency passband comes close to the cutoff passband and pushes it toward lower frequencies. The decrease of the cutoff frequency affects in turn the Fabry – Perot passband. The frequency of the chromospheric Fabry – Perot passband is in between 5.3 and 5.4 mHz. But this band is absent in the middle of the sunspot, where it is likely united with the cutoff frequency. Thus, there is a variation of the sunspot atmosphere across the sunspot.

In addition to Figures 4 and 5(a), Figures 5(b) – 5(d) display the amplitude spectrum of the sunspot oscillations at three positions along the slit. The spectrum in Figure 5(b) (where the location on the slit is at $1.3''$) looks perfect from the point of view of the filter theory. The Fabry – Perot passband is located on the low-frequency slope of the cutoff passband. On the high-frequency side of the cutoff a series of equidistant high-frequency interference passbands trace the exponential decline of the amplitude spectrum of the incident slow-wave noise, which is filtered by the multipassband atmospheric filter. The ideal equidistant location of the passbands can appear only in the case when the atmospheric layers responsible for the passbands formation have the same travel time as that of the slow waves across them. The quite different spectrum in Figure 5(c) is in the middle of the slit where there is the decrease of the frequency of the cutoff frequency. In this case the atmospheric layers do not have the same travel times across them. This makes the high-frequency passbands not equidistant and not equal in their transmission. Even the first high-frequency passband dominates over the cutoff passband. The number of high-frequency passbands increases from six to seven and the frequency difference between the adjacent passbands decreases. On the opposite side of the umbra (see Figure 5(d)) the number of high-frequency passbands decreases again to six, and they are far from being equidistant. It is clearly seen in Figure 5(d) that the

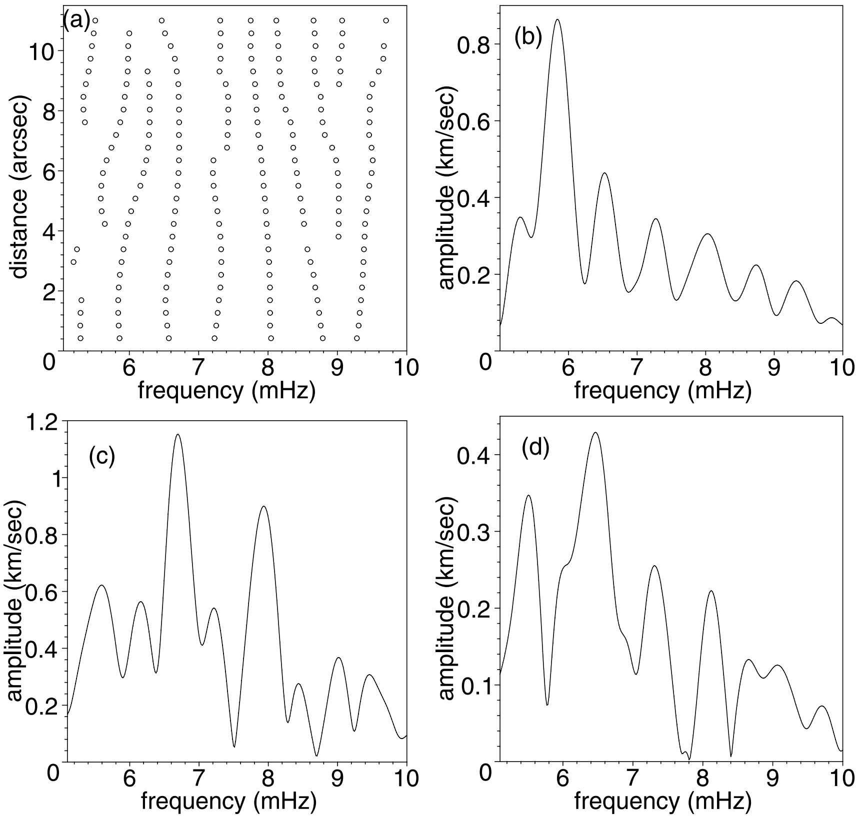

Figure 5 (a) The dependence of the peak frequencies in the spectrum as a function of distance along the slit. (b, c, d) The amplitude spectrum of the oscillations for the three positions on the slits: 1.3″ (b), 5.5″ (c), 11″ (d).

first high-frequency passband has merged with the cutoff passband and it is located on the low-frequency slope of the first high-frequency passband. This is why the frequency of the cutoff passband is not shown in Figure 5(a) for the far end of the slit. Thus, the variation of the spectrum of the sunspot oscillations across the umbra is revealed. This is a manifestation of the sunspot atmospheric variation across the umbra.

The exploration of empirical atmospheres reveals that the three layers have to be involved in the formation the high-frequency passbands. The number of high-frequency passbands and the mean frequency interval between them depends on the travel time of the slow waves through the entire sunspot atmosphere since all the atmospheric layers are involved in producing the interference effects. It is no surprise that the travel time through the atmosphere is less in the middle of the sunspot, since the central part of the sunspot is most likely cooler than peripheral parts. The distinction between the regular pattern of the equidistant high-frequency passbands on the one side of the sunspot (Figure 5(b)) and nonequidistant passbands on the other side (Figure 5(d)) is evidence that there are variations in some of the layers of the sunspot atmosphere. Therefore it comes as no surprise that the atmosphere varies across the sunspot, and it cannot be described satisfactory by the mean empirical model.

6. Discussion

The exploration of sunspot oscillations across the sunspot umbra reveals significant changes of the spectrum. This complicated behavior of the oscillations cannot be explained within the framework of the normal mode theory. Thus, this is one more argument for the filter theory, which can explain the multitude of effects appearing because of the interference of slow waves in the multilayer sunspot atmosphere. In connection with this the papers by Brynildsen *et al.* (2003) and Shibasaki (2001) should be mentioned. They explored sunspot oscillations, but because of technical problems time series of only 20-minute duration were available for their analysis. Such short time series do not permit the resolution of the structure of the spectrum – only one peak appears in the spectrum. But the authors of these papers ignored this point and made strong statements about the nature of sunspot oscillations and the occurrence of the chromospheric resonance. Unresolved spectra of sunspot oscillations cannot be used for the exploration of the nature of sunspot oscillations.

The revised version of the filter theory of the sunspot oscillations makes it possible to use the rich information about the structure of the sunspot atmosphere contained in the spectrum of three-minute oscillations. The observations provide solid proof that the sunspot atmosphere is nonuniform. The analysis of the observations presented here is just an example of how to use the filter theory for the exploration of sunspots. It is necessary to explore different sunspots and to use the observations of the oscillations of the entire sunspot.

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